

1

1B	$A + X \rightarrow 2X$ $X \rightarrow Y$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - x - xy$ $\dot{y} = x + xy - y$	$J = \begin{bmatrix} a - 1 - y & -x \\ 1 + y & x - 1 \end{bmatrix}$
$0; 0$	$J_1 = \begin{bmatrix} a - 1 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J_1 = 1 - a$ $\text{tr} J_1 = a - 2$	
$\frac{a - 1}{a}; a - 1$	$J_2 = \begin{bmatrix} 0 & (1 - a)/a \\ a & -1/a \end{bmatrix}$	$\det J_2 = a - 1$ $\text{tr} J_2 = -1/a$	

$a = 5$

$0; 0$	$\begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 3$	nyereg
$0,8; 4$	$\begin{bmatrix} 0 & -0,8 \\ 5 & -0,2 \end{bmatrix}$	$\det J = 4$ $\text{tr} J = -0,2$	stabil fókusz

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1B	$A + X \rightarrow 2X$ $X \rightarrow Y$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - x - xy$ $\dot{y} = x + xy - y$	$J = \begin{bmatrix} a - 1 - y & -x \\ 1 + y & x - 1 \end{bmatrix}$
$0; 0$	$J_1 = \begin{bmatrix} a - 1 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J_1 = 1 - a$ $\text{tr} J_1 = a - 2$	
$\frac{a - 1}{a}; a - 1$	$J_2 = \begin{bmatrix} 0 & (1 - a)/a \\ a & -1/a \end{bmatrix}$	$\det J_2 = a - 1$ $\text{tr} J_2 = -1/a$	

$a = -1$

$0; 0$	$\begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = -3$	stabil csomó
$2; -2$	$\begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}$	$\det J = -2$ $\text{tr} J = 1$	nyereg

3

1B	$A + X \rightarrow 2X$ $X \rightarrow Y$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - x - xy$ $\dot{y} = x + xy - y$	$J = \begin{bmatrix} a - 1 - y & -x \\ 1 + y & x - 1 \end{bmatrix}$
$0; 0$	$J_1 = \begin{bmatrix} a - 1 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J_1 = 1 - a$ $\text{tr} J_1 = a - 2$	
$\frac{a - 1}{a}; a - 1$	$J_2 = \begin{bmatrix} 0 & (1 - a)/a \\ a & -1/a \end{bmatrix}$	$\det J_2 = a - 1$ $\text{tr} J_2 = -1/a$	

$a = -2$

$0; 0$	$\begin{bmatrix} -3 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J = 3$ $\text{tr} J = -4$	stabil csomó
$1,5; -3$	$\begin{bmatrix} 0 & -1,5 \\ -2 & 0,5 \end{bmatrix}$	$\det J = -3$ $\text{tr} J = 0,5$	nyereg

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1C	$A + X \rightarrow 2X$ $2X \rightarrow 2Y$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - 2x^2 - xy$ $\dot{y} = 2x^2 + xy - y$	$J = \begin{bmatrix} a - 4x - y & -x \\ 4x + y & x - 1 \end{bmatrix}$
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$0; 0$	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$
$\frac{a}{a+2}; \frac{a^2}{a+2}$	$J_2 = \begin{bmatrix} -2a/(a+2) & -a/(a+2) \\ a(4+a)/(a+2) & -2/(a+2) \end{bmatrix}$	$\det J_2 = a$ $\text{tr} J_2 = -2(a+1)/(a+2)$

$a = 2$

$0; 0$	$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -2$ $\text{tr} J = 1$	nyereg
$0,5; 1$	$\begin{bmatrix} -1 & -0,5 \\ 3 & -0,5 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = -1,5$	stabil fókusz

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1C	$\begin{aligned} A + X &\rightarrow 2X \\ 2X &\rightarrow 2Y \\ X + Y &\rightarrow 2Y \\ Y &\rightarrow \end{aligned}$	$\begin{aligned} \dot{x} &= ax - 2x^2 - xy \\ \dot{y} &= 2x^2 + xy - y \end{aligned}$	$J = \begin{bmatrix} a - 4x - y & -x \\ 4x + y & x - 1 \end{bmatrix}$
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$0; 0$	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{aligned} \det J_1 &= -a \\ \text{tr} J_1 &= a - 1 \end{aligned}$
$\frac{a}{a+2}; \frac{a^2}{a+2}$	$J_2 = \begin{bmatrix} -2a/(a+2) & -a/(a+2) \\ a(4+a)/(a+2) & -2/(a+2) \end{bmatrix}$	$\begin{aligned} \det J_2 &= a \\ \text{tr} J_2 &= -2(a+1)/(a+2) \end{aligned}$

$a = -3$

$0; 0$	$\begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{aligned} \det J &= 3 \\ \text{tr} J &= -4 \end{aligned}$	stabil csomó
$3; -9$	$\begin{bmatrix} -6 & -3 \\ 3 & 2 \end{bmatrix}$	$\begin{aligned} \det J &= -3 \\ \text{tr} J &= -4 \end{aligned}$	nyereg

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1C	$A + X \rightarrow 2X$ $2X \rightarrow 2Y$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - 2x^2 - xy$ $\dot{y} = 2x^2 + xy - y$	$J = \begin{bmatrix} a - 4x - y & -x \\ 4x + y & x - 1 \end{bmatrix}$
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$0; 0$	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$
$\frac{a}{a+2}; \frac{a^2}{a+2}$	$J_2 = \begin{bmatrix} -2a/(a+2) & -a/(a+2) \\ a(4+a)/(a+2) & -2/(a+2) \end{bmatrix}$	$\det J_2 = a$ $\text{tr} J_2 = -2(a+1)/(a+2)$

$a = 3$

$0; 0$	$\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -3$ $\text{tr} J = 2$	nyereg
$0,6; 1,8$	$\begin{bmatrix} -1,2 & -0,6 \\ 4,2 & -0,4 \end{bmatrix}$	$\det J = 3$ $\text{tr} J = -1,6$	stabil fókusz

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7A	$A + 2X \rightarrow 3X$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax^2 - xy$ $\dot{y} = xy - y$	$J = \begin{bmatrix} 2ax - y & -x \\ y & x - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = 0$ $\text{tr} J_1 = -1$	
1; a	$J_2 = \begin{bmatrix} a & -1 \\ a & 0 \end{bmatrix}$	$\det J_2 = a$ $\text{tr} J_2 = a$	

a = 2

0; 0	$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = 0$ $\text{tr} J = -1$	stabil nyeregcsomó
1; 2	$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = 2$	instabil fókusz

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7A	$A + 2X \rightarrow 3X$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax^2 - xy$ $\dot{y} = xy - y$	$J = \begin{bmatrix} 2ax - y & -x \\ y & x - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = 0$ $\text{tr} J_1 = -1$	
1; a	$J_2 = \begin{bmatrix} a & -1 \\ a & 0 \end{bmatrix}$	$\det J_2 = a$ $\text{tr} J_2 = a$	

$a = -1$

0; 0	$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = 0$ $\text{tr} J = -1$	stabil nyeregcsomó
1; -1	$\begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}$	$\det J = -1$ $\text{tr} J = -1$	nyereg

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7A	$A + 2X \rightarrow 3X$ $X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax^2 - xy$ $\dot{y} = xy - y$	$J = \begin{bmatrix} 2ax - y & -x \\ y & x - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = 0$ $\text{tr} J_1 = -1$	
1; a	$J_2 = \begin{bmatrix} a & -1 \\ a & 0 \end{bmatrix}$	$\det J_2 = a$ $\text{tr} J_2 = a$	

a = 5

0; 0	$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = 0$ $\text{tr} J = -1$	stabil nyeregcsomó
1; 5	$\begin{bmatrix} 5 & -1 \\ 5 & 0 \end{bmatrix}$	$\det J = 5$ $\text{tr} J = 5$	instabil csomó

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3A	$A + X \rightarrow 2X$ $X + 2Y \rightarrow 3Y$ $Y \rightarrow$	$\dot{x} = ax - xy^2$ $\dot{y} = xy^2 - y$	$J = \begin{bmatrix} a - y^2 & -2xy \\ y^2 & 2xy - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$	
$1/\sqrt{a}; \sqrt{a}$ $-1/\sqrt{a}; -\sqrt{a}$	$J_2 = J_3 = \begin{bmatrix} 0 & -2 \\ a & 1 \end{bmatrix}$	$\det J_{2,3} = 2a$ $\text{tr} J_{2,3} = 1$	

a = 1

0; 0	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -1$ $\text{tr} J = 0$	nyereg
1; 1	$\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = 1$	instabil fókusz
-1; -1	$\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = 1$	instabil fókusz

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3A	$A + X \rightarrow 2X$ $X + 2Y \rightarrow 3Y$ $Y \rightarrow$	$\dot{x} = ax - xy^2$ $\dot{y} = xy^2 - y$	$J = \begin{bmatrix} a - y^2 & -2xy \\ y^2 & 2xy - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$	
$1/\sqrt{a}; \sqrt{a}$ $-1/\sqrt{a}; -\sqrt{a}$	$J_2 = J_3 = \begin{bmatrix} 0 & -2 \\ a & 1 \end{bmatrix}$	$\det J_{2,3} = 2a$ $\text{tr} J_{2,3} = 1$	

a = 4

0; 0	$\begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 0$	nyereg
0,5; 2	$\begin{bmatrix} 0 & -2 \\ 4 & 1 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = 1$	instabil fókusz
-0,5; -2	$\begin{bmatrix} 0 & -2 \\ 4 & 1 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = 1$	instabil fókusz

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3B	$A + X \rightarrow 2X$ $X \rightarrow Y$ $X + 2Y \rightarrow 3Y$ $Y \rightarrow$	$\dot{x} = ax - x - xy^2$ $\dot{y} = x + xy^2 - y$	$J = \begin{bmatrix} a - 1 - y^2 & -2xy \\ 1 + y^2 & 2xy - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a - 1 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J_1 = 1 - a$ $\text{tr} J_1 = a - 2$	
$\frac{\sqrt{a-1}}{a}; \frac{\sqrt{a-1}}{a}$ $-\frac{\sqrt{a-1}}{a}; -\frac{\sqrt{a-1}}{a}$	$J_2 = J_3 = \begin{bmatrix} 0 & -2(a-1)/a \\ a & (a-2)/a \end{bmatrix}$	$\det J_{2,3} = 2(a-1)$ $\text{tr} J_{2,3} = (a-2)/a$	

a = 5

0; 0	$\begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 3$	nyereg
0,4; 2	$\begin{bmatrix} 0 & -1,6 \\ 5 & 0,6 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = 0,6$	instabil fókusz
-0,4; -2	$\begin{bmatrix} 0 & -1,6 \\ 5 & 0,6 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = 0,6$	instabil fókusz

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3B	$A + X \rightarrow 2X$ $X \rightarrow Y$ $X + 2Y \rightarrow 3Y$ $Y \rightarrow$	$\dot{x} = ax - x - xy^2$ $\dot{y} = x + xy^2 - y$	$J = \begin{bmatrix} a - 1 - y^2 & -2xy \\ 1 + y^2 & 2xy - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a - 1 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J_1 = 1 - a$ $\text{tr} J_1 = a - 2$	
$\sqrt{a-1}/a; \sqrt{a-1}$ $-\sqrt{a-1}/a; -\sqrt{a-1}$	$J_2 = J_3 = \begin{bmatrix} 0 & -2(a-1)/a \\ a & (a-2)/a \end{bmatrix}$	$\det J_{2,3} = 2(a-1)$ $\text{tr} J_{2,3} = (a-2)/a$	

a = 10

0; 0	$\begin{bmatrix} 9 & 0 \\ 1 & -1 \end{bmatrix}$	$\det J = -9$ $\text{tr} J = 8$	nyereg
0,3; 3	$\begin{bmatrix} 0 & -1,8 \\ 10 & 0,8 \end{bmatrix}$	$\det J = 18$ $\text{tr} J = 0,8$	instabil fókusz
-0,3; -3	$\begin{bmatrix} 0 & -1,8 \\ 10 & 0,8 \end{bmatrix}$	$\det J = 18$ $\text{tr} J = 0,8$	instabil fókusz

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5A	$A + X \rightarrow 2X$ $2X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - 2x^2y$ $\dot{y} = x^2y - y$	$J = \begin{bmatrix} a - 4xy & -2x^2 \\ 2xy & x^2 - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$	
1; a/2 -1; -a/2	$J_2 = J_3 = \begin{bmatrix} -a & -2 \\ a & 0 \end{bmatrix}$	$\det J_{2,3} = 2a$ $\text{tr} J_{2,3} = -a$	

a = 4

0; 0	$\begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 3$	nyereg
1; 2	$\begin{bmatrix} -4 & -2 \\ 4 & 0 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = -4$	stabil fókusz
-1; -2	$\begin{bmatrix} -4 & -2 \\ 4 & 0 \end{bmatrix}$	$\det J = 8$ $\text{tr} J = -4$	stabil fókusz

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5A	$A + X \rightarrow 2X$ $2X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - 2x^2y$ $\dot{y} = x^2y - y$	J $= \begin{bmatrix} a - 4xy & -2x^2 \\ 2xy & x^2 - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$	
1; a/2 -1; -a/2	$J_2 = J_3 = \begin{bmatrix} -a & -2 \\ a & 0 \end{bmatrix}$	$\det J_{2,3} = 2a$ $\text{tr} J_{2,3} = -a$	

a = 10

0; 0	$\begin{bmatrix} 10 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = -10$ $\text{tr} J = 9$	nyereg
1; 5	$\begin{bmatrix} -10 & -2 \\ 10 & 0 \end{bmatrix}$	$\det J = 20$ $\text{tr} J = -10$	stabil csomó
-1; -5	$\begin{bmatrix} -10 & -2 \\ 10 & 0 \end{bmatrix}$	$\det J = 20$ $\text{tr} J = -10$	stabil csomó

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5A	$A + X \rightarrow 2X$ $2X + Y \rightarrow 2Y$ $Y \rightarrow$	$\dot{x} = ax - 2x^2y$ $\dot{y} = x^2y - y$	J $= \begin{bmatrix} a - 4xy & -2x^2 \\ 2xy & x^2 - 1 \end{bmatrix}$
0; 0	$J_1 = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix}$	$\det J_1 = -a$ $\text{tr} J_1 = a - 1$	
1; a/2 -1; -a/2	$J_2 = J_3 = \begin{bmatrix} -a & -2 \\ a & 0 \end{bmatrix}$	$\det J_{2,3} = 2a$ $\text{tr} J_{2,3} = -a$	

a = -2

0; 0	$\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$	$\det J = 2$ $\text{tr} J = -3$	stabil csomó
1; -1	$\begin{bmatrix} 2 & -2 \\ -2 & 0 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 2$	nyereg
-1; 1	$\begin{bmatrix} 2 & -2 \\ -2 & 0 \end{bmatrix}$	$\det J = -4$ $\text{tr} J = 2$	nyereg